# Experimental control of nonlinear dynamics by slow parametric modulation

Alexander N. Pisarchik,*,† V. N. Chizhevsky,*,† and Ramón Corbalán
*Departament de Física, Universitat Autònoma de Barcelona, E-08193 Bellaterra, Barcelona, Spain*

Ramon Vilaseca
*Departament de Física i Enginyeria Nuclear, Universitat Politècnica de Catalunya, Colom 11, E-08222 Terrassa, Spain*

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We provide experimental evidence that a slow parametric modulation can control dynamical regimes and inhibit chaos in a nonlinear system. We demonstrate this effect in a loss-modulated $CO_2$ laser. We show the influence of the amplitude and frequency of the control modulation (in our case, the cavity detuning) on the efficiency of this method of nonfeedback control. [S1063-651X(97)04903-9]

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## I. INTRODUCTION

Since the appearance of the work of Ott, Grebogi, and Yorke [1], who proposed a method of controlling chaos, an active search for methods of chaos suppression in a large number of dynamical systems has been conducted. Many methods are based on the stabilization of unstable periodic orbits embedded within a chaotic attractor. This can be realized by applying *feedback* control to an available system parameter [1–3] or by periodic modulation of one of the system parameters at the appropriate frequency [4–6], which is known as *nonfeedback* control. Although the methods of nonfeedback control are generally not as effective as the feedback methods, they do not require prior knowledge of the system behavior. Therefore, they are particularly appealing for systems whose state is impossible or difficult to measure in real time and where feedback control is very hard to realize (e.g., some kinds of biological or chemical processes).

The efficiency of nonfeedback control is known [4,6] to depend strongly on the frequency of the control modulation. At the resonant frequency (generally, this means that the ratio between the control frequency and a characteristic frequency of the system is a rational number) a small parametric perturbation is able to bring the system to a regular regime [4,7–9], while for chaos suppression by nonresonant (or near-resonant) modulation a relatively large perturbation amplitude is required [10]. In actual practice the nonresonant control is more convenient because it does not require the exact measurement of the characteristic frequency, but as a counterpart the perturbation amplitude has to be larger than in the resonant case. Recently, it was theoretically shown that nonresonant parametric perturbations at either high [11] or low [12] frequencies can also stabilize chaos. In the latter case Vilaseca *et al.* [12] have numerically shown that the accurate stabilization of an unstable steady state in an autonomous system can be achieved by large-amplitude slow modulation (in comparison with the characteristic frequency) of a control parameter. They have also made the suggestion about the possibility of taming chaos in a nonautonomous system by a nonfeedback slow modulation. The physical mechanism of this phenomenon lies in the variation of the conditions so that the system passes back and forth through an instability point.

In this work we present experimental evidence of controlling nonlinear dynamics by large-amplitude slow nonresonant parametric modulation in a nonautonomous system, namely, in a $CO_2$ laser with modulated losses. In contrast with control methods that use small-amplitude perturbations and do not modify the shape of the stabilized cycle, in our method the state of system becomes slowly modulated at the control frequency. As distinct from an autonomous system, the characteristic frequency in our $CO_2$ laser is determined by an external modulation of the cavity losses. The applicability of the slow modulation technique for controlling nonlinear dynamics in such a system is based on the combined effect of two well-known features of a modulated class-B laser (such as our $CO_2$ laser). They are (i) the existence of a minimum in the period-doubling instability boundaries near the relaxation oscillation frequency [13,14] and (ii) a delay of the bifurcation when the control parameter is swept through the instability [15]. Let us consider in greater detail each of these features.

(i) The first feature was observed by Tredicce *et al.* [13] in a $CO_2$ laser with a modulated parameter at the driving frequency $f_0$. They found some inverted resonances in the amplitude $V_0$ of the driving signal versus the driving frequency [see Fig. 1(a)], at which the period-doubling boundaries are located at the minimal driving amplitude. Due to the interaction between the driving frequency and the relaxation oscillation frequency $f_r$, one such resonance occurs when the driving frequency matches the frequency of the relaxation oscillations ($f_0=f_r$). It is known (see, for example, [16]) that $f_r$ depends on the cavity losses and gain. Thus, varying the laser gain (by changing, for instance, the cavity detuning), one can choose the operating point in the phase diagram. For example, if the operating point is chosen at the resonance ($f_0=f_r$) in the chaotic region ($V_0=V^{\mathrm{ch}}$), one can get a bifurcation diagram involving a sequence of period doublings (i.e., $T$-$2T$-$4T$-$\cdots$-chaos) followed by the inverse process (chaos-$\cdots$-$4T$-$2T$-$T$), varying quasistatically the

*Permanent address: B. I. Stepanov Institute of Physics, Belarus Academy of Sciences, 220072 Minsk, Belarus.
†FAX: +34-3-5812155. Electronic address: corbalanr@cc.uab.es

 

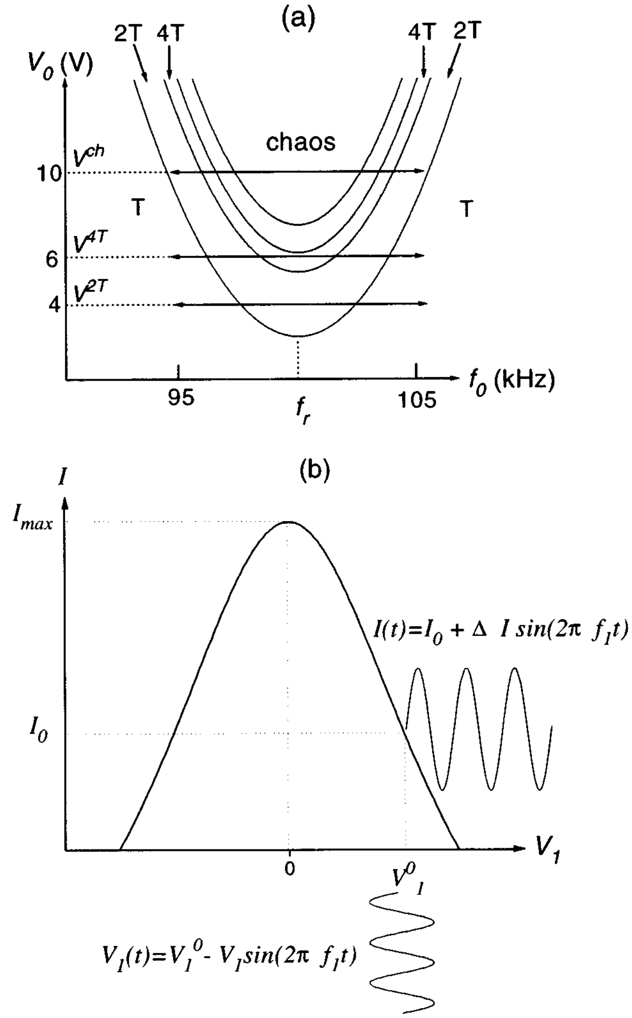


FIG. 1. (a) Schematic phase diagram in the parameter space. The operating point can be chosen by varying the driving amplitude $V_0$ to be equal to $V^{2T}$ ($2T$ regime), $V^{4T}$ ($4T$ regime), or $V^{\mathrm{ch}}$ (chaotic regime). (b) Schematic of steady-state laser intensity versus control signal. The picture illustrates the choice of the operating point (at $V_1^0$ and $I_0$). The periodic modulation of the cavity detuning at the voltage $V_1$ applied to the piezoceramic leads to the appropriate modulation of the laser intensity $I$ at the frequency $f_1$.

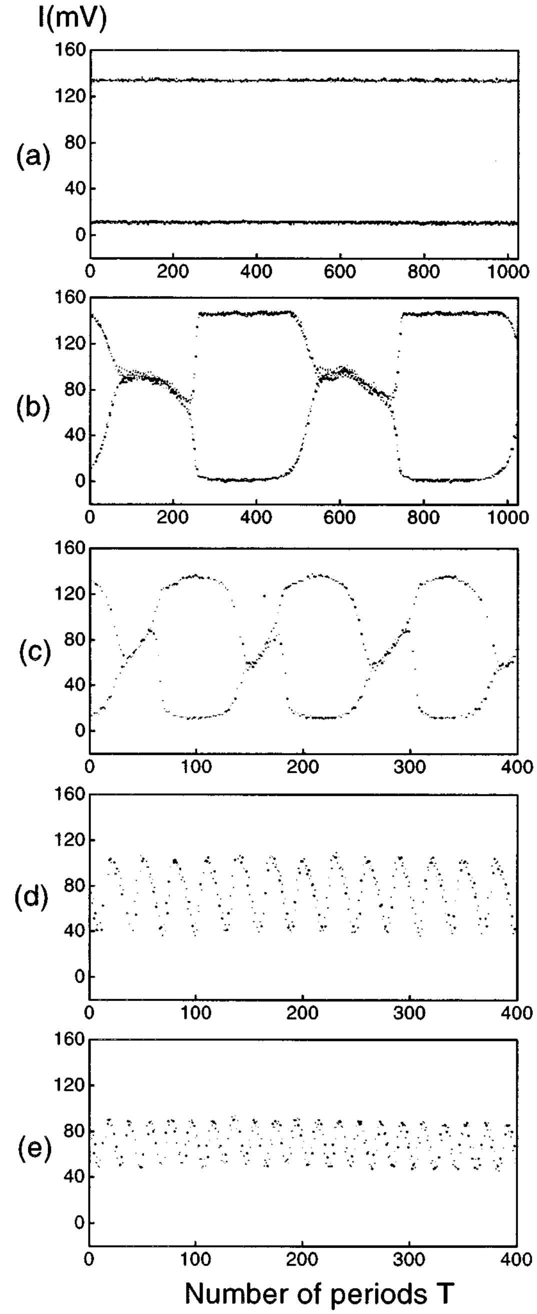


FIG. 2. Stroboscopic measurements of laser intensity at different control frequencies. (a) Without control modulation the laser operates in a $2T$ regime. (b) $f_1=200$ Hz. (c) $f_1=850$ Hz. (d) $f_1=3$ kHz. (e) $f_1=5$ kHz. $V_0=4$ V. The modulation depth $M\approx 40\%$.

driving frequency near the relaxation oscillation frequency or the relaxation oscillation frequency near the driving frequency. Such diagrams have been called ''bubbles'' or ''period bubbling'' [17,18].

(ii) It is known that near the onset of an instability the system needs a relatively large time to reach a steady state [15]. When the sweeping rate of the control parameter is increased, the bifurcation diagram exhibits a dynamic deformation. The postponement of bifurcations on the bifurcation diagram due to the sweeping rate of the control parameter was experimentally observed in many systems (e.g., in an electrical circuit [19] and lasers [20,21]) and was theoretically treated by Mandel *et al.* [15].

In our experiments we combine both of these effects: keeping the driving frequency close to the relaxation oscillation frequency (i.e., close to the resonance), we slowly modulate the relaxation oscillation frequency to get bubbles by changing the cavity length. In such a manner we cause the laser to pass through the boundaries of period doublings in the parameter space [see Fig. 1(a)]. The proper choice of the control frequency allows one to postpone the bifurcations so that the delay time becomes longer that the half period of the slow modulation and some bifurcations are ''passed over'' or scrambled because of dynamical effects [22]. Thus we can say that the slow periodic modulation stabilizes the system. We should add that even when the initial point is chosen out of the resonance (i.e., when $f_0\neq f_r$), the slow modulation of the relaxation oscillation frequency would provide a similar effect if the system passes through instability points, but with lower efficiency.

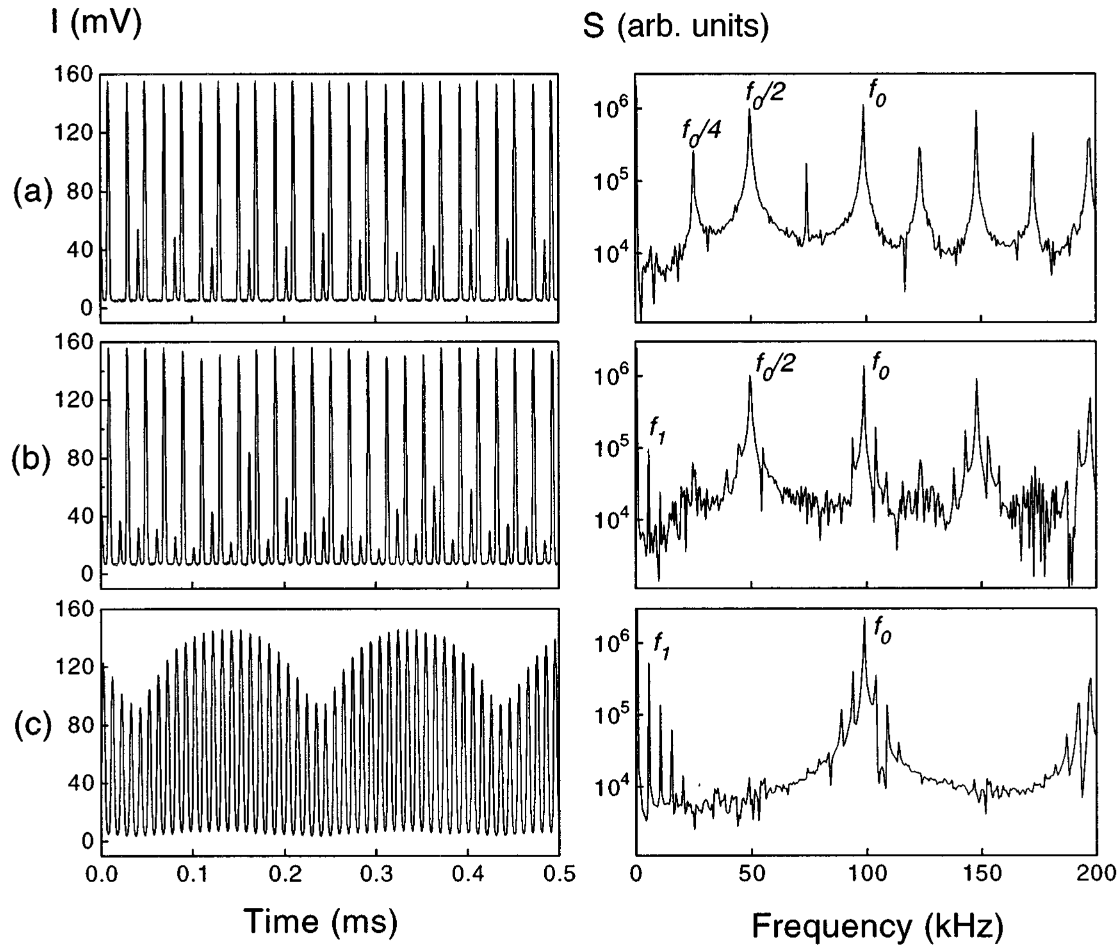


FIG. 3. (a) $4T$-periodic time trace and corresponding power spectrum stabilized with slow modulation to (b) the $2T$ regime (at $V_1=1.5$ V, $M\approx 15\%$) and to (c) the $T$ regime (at $V_1=3$ V, $M\approx 40\%$). $f_1=5$ kHz. $V_0=6$ V. The frequency $f_1$ and its satellites appear in the spectra.

The rest of the paper is organized as follows. In Sec. II we describe the experimental setup. In Sec. III we show how the slow parametric modulation decreases the periodicity of the system and investigate the influence of the control frequency. In Sec. IV we demonstrate with experimental time series and power spectra the stabilizing effect of the slow modulation at different modulation amplitudes on the weakly chaotic system. Finally, conclusions are given in Sec. V.

## II. EXPERIMENTAL SETUP

The experiments have been performed on a single-mode $CO_2$ laser with modulated losses via an elasto-optic KRS-5 modulator inserted in the laser cavity. The experimental arrangement is similar to that described in previous works [23]. An electric signal is applied to the modulator providing the time-dependent cavity losses. This signal (the driving one for our system) $V_0\sin(2\pi f_0 t)$ has a frequency $f_0=99$ kHz and an amplitude $V_0$, which provides the choice of the appropriate operating point in the parameter space (i.e., $2T$, $4T$, or chaotic regime); see Fig. 1(a). The frequency of the relaxation oscillations of our laser is approximately 100 kHz.

The output laser intensity is detected with a $Cd_xHg_{1-x}Te$ detector and displayed on a Tektronix DSA 602A digitizing signal analyzer that performs the power Fourier transform of the signal. Stroboscopic measurements are carried out with a Tektronix 2440 digital oscilloscope using the period $T=1/f_0$ of the loss modulation as an external clock for sampling the laser intensity.

The control signal $V_1(t)=V_1^0+V_1\sin(2\pi f_1 t)$ is applied to the piezotranslator that tunes the output mirror. This signal produces the appropriate changes in the cavity detuning, which are proportional to the constant component $V_1^0$ and to the alternative component $V_1\sin(2\pi f_1 t)$ of the signal voltage. A theoretical description of the impact of detuning on a single-mode modulated laser can be found in several works (see, for example, [24]).

To clarify the situation under consideration we show schematically in Fig. 1(b) the Lorenz shape of the laser gain in the units of the steady-state laser intensity $I$ vs the voltage $V_1^0$ applied to the piezotranslator. With the constant component $V_1^0$ we choose the initial point (at $I=I_0$) to be out of the resonance, in the domain where the intensity depends approximately linearly on the detuning. Thus periodic modulation of the cavity length (with amplitude $V_1$) leads to periodic modulation of the laser intensity (with amplitude $\Delta I$) and hence of the relaxation oscillation frequency (in the

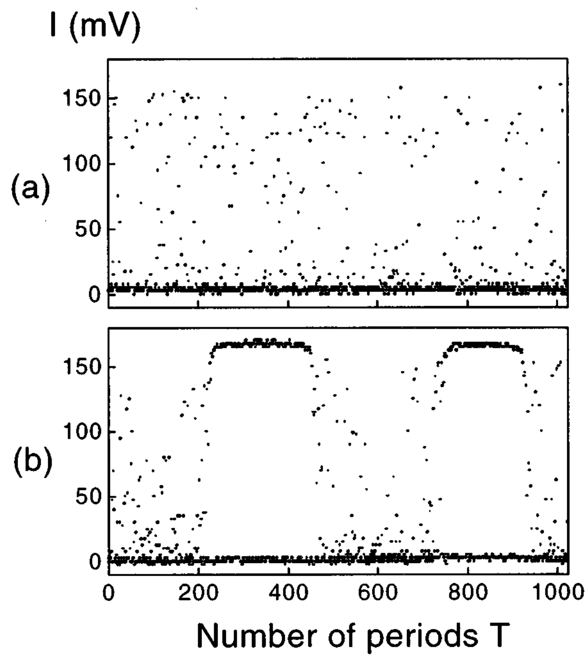


FIG. 4. Stroboscopic measurements that represent the transformation of chaos to periodically alternated $2T$ periodic and chaotic motions. (a) Without control modulation. (b) With control modulation at $f_1=200$ Hz, $M\approx 15\%$. $V_0=10$ V.

range of about $\pm 5$ kHz) [16]. As a consequence, the whole phase diagram of Fig. 1(a) oscillates at the control frequency $f_1$ in the parameter space, in the horizontal direction, near the initial point, i.e., near the driving frequency $f_0$. In this condition, the system crosses back and forth the instability boundaries.

Since the response of the piezoelectric ceramic depends on the modulation frequency $f_1$, we introduce the modulation depth $M=2\Delta I/I_0$. This allows one to compare the stabilization effect at different signal frequencies.

## III. STABILIZATION OF PERIODIC ORBITS

In this section we study the effect of the slow parametric modulation on the laser response when without this modulation the laser operates in a periodic regime. In other words, by changing the amplitude $V_0$, we first select the operating point in the phase diagram in order to obtain, for instance, a $2T$ regime (at $V_0=4$ V) [see Fig. 1(a)]. Next, we apply the slow modulation of the cavity detuning. As a result, the phase diagram moves forward and backward with the frequency $f_1$ and the laser dynamics involves successively $2T$ and $T$ regimes.

In Fig. 2 we show several stroboscopic diagrams obtained at different control frequencies $f_1$. Without modulation ($V_1=0$) the laser operates in a $2T$ regime [see Fig. 2(a)]. With slow but relatively large modulation (the modulation depth $M\approx 0.4$) the ''bubbles'' appear [see Figs. 2(b) and 2(c)], i.e., the $2T$- and $T$-periodic regimes successively alternate during one period of the control modulation. On further increasing $f_1$, the ''bubbles'' disappear and only a $T$-periodic regime is obtained [see Figs. 2(d) and 2(e)], which is modulated at the control frequency.

In our experiments we observed dynamical bistability effects even at very low sweeping rate of the control parameter (at several hertz). These effects manifest themselves in the fact that the transition between $T$ and $2T$ regimes and the reverse transition are different [see Figs. 2(b) and 2(c)] because in one case the perturbation $V_1(t)$ is increasing while in the other it is decreasing. As indicated in [20], the width of the dynamically induced ''bistable'' region in the bifurcation diagrams at forward and backward sweeping of the control parameter varies as the square root of the rate of change of the bifurcation parameter in accordance with the prediction of Mandel *et al.* [15] on a nonautonomous quadratic map. With increasing modulation frequency, the period-doubling bifurcation is postponed so that the bifurcation does not appear during the half period of the control modulation. Thus there is a lowest limit of the control frequency where the period bubbling transforms to a single-periodical movement. In our case this limit frequency is equal to about 2 kHz.

Figure 3 illustrates the suppression of an initial $4T$ regime (at $V_0=6$ V) with the time series and corresponding power spectra. The initial $4T$ regime at $V_1=0$ V is shown in Fig. 3(a). Increasing the control amplitude to $V_1=1.5$ V, the $4T$-periodic regime disappears and the system transfers to $2T$-periodic regime [see Fig. 3(b)]; then, with further increasing $V_1$ to 3 V, the $2T$ regime also disappears and only period-$T$ remains [see Fig. 3(c)], which is slowly modulated at $f_1$. It is clearly seen that with increasing $V_1$, the frequency $f_1$ and difference frequencies appear in the spectra [see Figs. 3(b) and 3(c)]. Thus, choosing the operation points at the resonance ($f_r=f_0$) and starting from one subharmonic frequency (e.g., at $V_0=V^{2T}$ or $V^{4T}$) [see Fig. 1(a)], we can stabilize the system over different periodic regimes by changing the amplitude $V_1$ of the control signal.

## IV. INHIBITION OF CHAOS

Let us consider the effect of the slow modulation of the detuning in the case where the initial state is chaotic [$V_0=V^{\mathrm{ch}}=10$ V in Fig. 1(a)]. In deciding on the operation point, the best conditions for chaos suppression are achieved when the control parameter, during its excursion, crosses all the period-doubling bifurcation boundaries [12]. In our case, since we have chosen the cavity detuning (or the voltage applied to the piezotranslator) as a control parameter, the amplitude of the modulation is restricted by the half-width of the gain shape [see Fig. 1(b)]. This imposes a limitation on the dynamic range of the variation of the relaxation oscillation frequency. Because of the small dynamic range we cannot eliminate chaos completely, but decrease essentially the ''degree of chaos,'' or complexity of the motion, in the system. We demonstrate this effect with time series and power spectra.

In Fig. 4 we show how the slow modulation influences the laser dynamics when the initial state is chaotic. Figure 4(a) displays the stroboscopic measurement of the laser intensity in the absence of the control modulation. In the presence of the control modulation, stable $2T$ regions appear in the laser output and alternate periodically with chaotic intervals. We have observed this periodic alternation with $f_1=$ 30, 200, 2000, and 5000 Hz. One stroboscopic diagram (for $f_1=200$

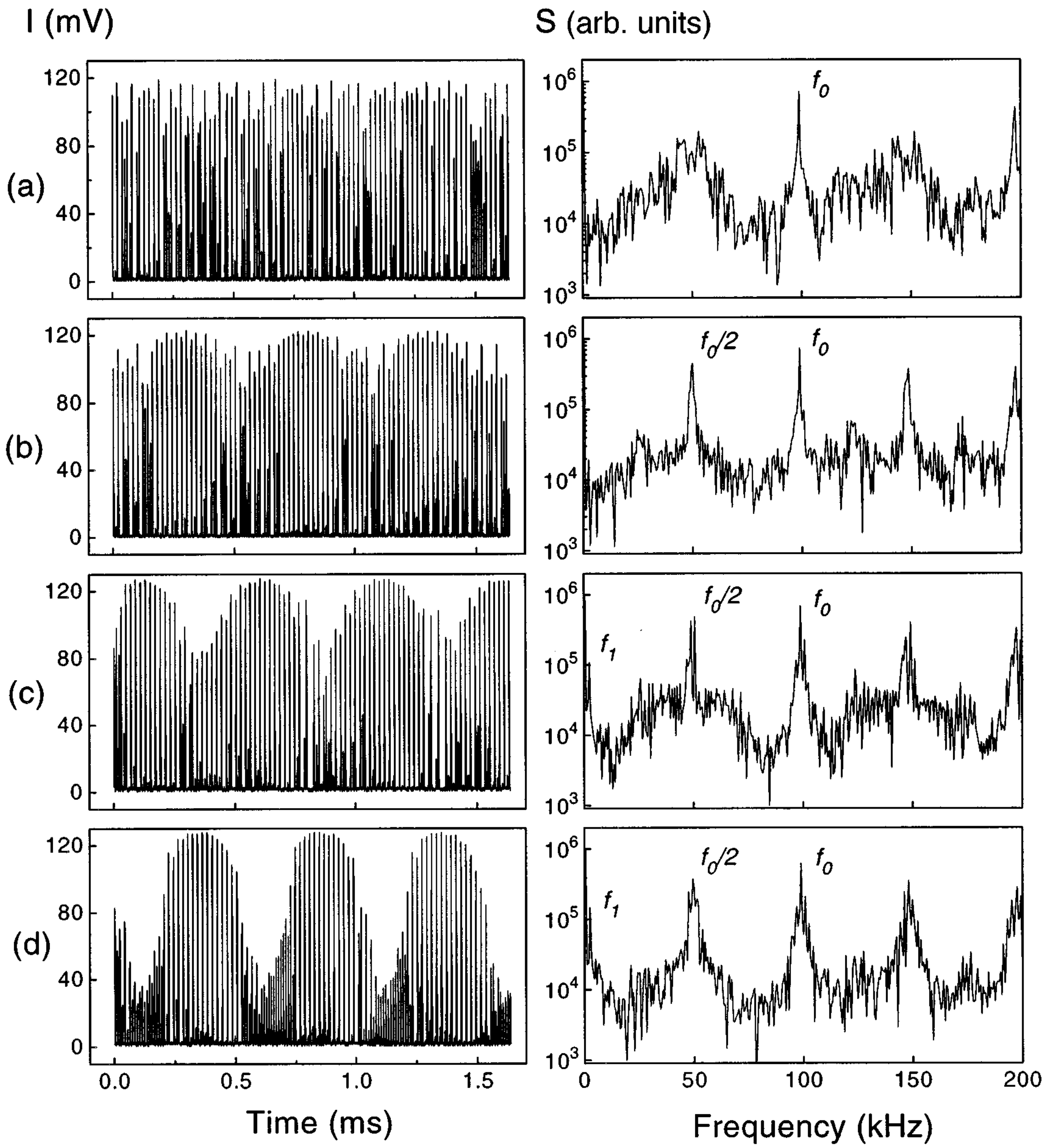


FIG. 5. Chaotic time traces and corresponding power spectra partially stabilized to the $2T$ regime at different modulation amplitudes $V_1$. (a) $V_1=0$. (b) $V_1=3$ V, $M\approx20\%$. (c) $V_1=6$ V, $M\approx35\%$. (d) $V_1=8$ V, $M\approx75\%$. $V_0=10$ V, $f_1=2$ kHz.

Hz) is shown in Fig. 4(b). It is remarkable that no other periodic cycles ($4T$, $8T$, etc.) are observed in the laser output at control frequency $f_1\geqslant30$ Hz. The main possible reason for the disappearance of the other periodic regimes is the small range of their existence in the phase diagram in comparison with $2T$-periodic and chaotic areas [Fig. 1(a)]. Due to the dynamical deformation they do not appear in the laser response.

In Fig. 5 we demonstrate with the chaotic time series and corresponding spectra the effect of modulation amplitude on the inhibition of chaos at the modulation frequency $f_1=2$ kHz. Before $V_1$ is applied, the power spectrum displays a broadband feature that is a characteristic of chaos [see Fig. 5(a)]. In the presence of the control modulation, one can see from the power spectra shown in Figs. 5(b)–5(d) that the level of the broadband noise decreases and that sharp lines at $f_0/2$ as well as the spikes at $f_1$ and its harmonics appear in the spectra. This indicates that the degree of chaos is reduced and the chaotic motion is partially converted to the $2T$-periodic regime. Increasing the amplitude of the control modulation up to 75% modulation [see Fig. 5(d)] does not allow the system to reach the $T$-periodic range in the parameter space because the cavity detuning is limited by the half-width of the laser gain shape [see Fig. 1(b)] and does not lead to a substantial change in the relaxation oscillation frequency. However, at very high control amplitude $M=100\%$ and frequency $f_1\geqslant5$ kHz both the $2T$-periodic and chaotic cycles almost disappear along with lasing. As a consequence, the laser operates in the pulsed regime with $T$-periodic cycles (Fig. 6). No other periodic regimes are observed either in the time domains or in the power spectra. Due to pulsed lasing, the spikes corresponding to the modulation frequency $f_1$, as can seen from Fig. 6(b), become very large.

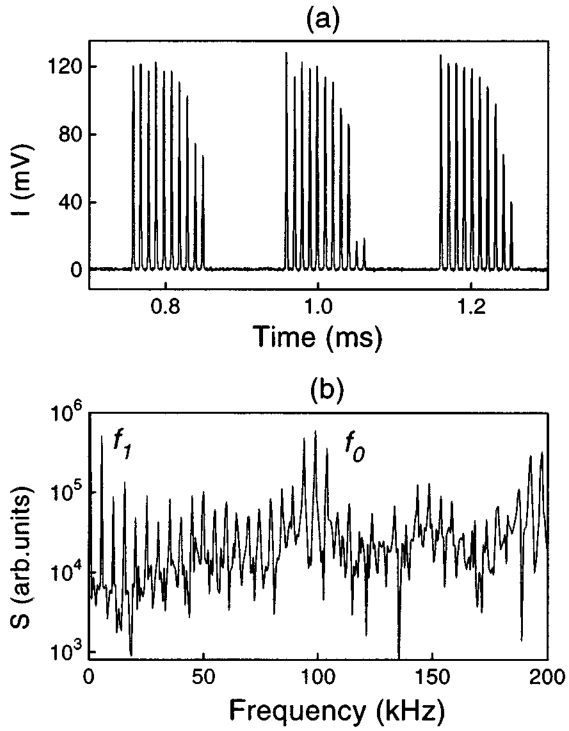


FIG. 6. (a) $T$-periodic pulses of lasing and (b) corresponding power spectrum at $f_1 = 5$ kHz and $V_1 = 10$ V. $V_0 = 10$ V.

## V. CONCLUSION

In this article we have shown that the idea of controlling nonlinear dynamics by a slow parametric modulation can successfully be applied to a nonautonomous system. We have demonstrated in experiments with a loss-modulated $CO_2$ laser that by means of a slow modulation of the cavity length one can control the nonlinear dynamics of the laser. This becomes possible due to the combined effect of the delay occurring in a swept bifurcation and the particular structure of the periodicity regions in the phase diagram of a $CO_2$ laser with modulated losses.

The addition of a slow parametric modulation is able to stabilize the periodic orbits or, in other words, to reduce the periodicity of a nonlinear system (e.g., from $2T$ to $T$ or from $4T$ to $2T$ and to T) when the initial state is periodic. The transformation of the dynamic state from one periodic behavior to another one can be performed by the proper choice of the modulation amplitude and frequency.

Although we have not been able to fully stabilize chaos, a certain inhibition of chaos has been demonstrated. In particular, we managed to transform the chaotic behavior to the periodically alternated $2T$ and chaotic motions or to the pulsed $T$-periodic regime. Other periodic regimes were not detected in the laser output. The disadvantage of the method used is a slow modulation of the laser output. Because of the narrow dynamic range of the modulation of the cavity detuning, limited by the half-width of the gain shape, we did not succeed in eliminating chaos completely and the driven laser did not reach the $T$-periodic regime. However, a similar approach, in our opinion, may be applied by modulating another parameter (for example, the discharge current) that probably will give more efficiency in chaos suppression.

Although our experiments have been performed with a laser, the approach employed in this work can be applied to many different nonlinear systems including autonomous ones. This method does not require any *a priori* knowledge of the system state or a feedback loop. The main principle implies a slow but relatively large external modulation of the control parameter. However, it should be noted that the efficiency of the stabilization depends on the amplitude and frequency of the control modulation. It is important to find optimal parameters in each concrete case.

Perhaps the method of stabilization by a slow modulation is already working in nature. Many self-organized dynamical systems have two parameters that are modulated at two different frequencies. One of them may be considered as a characteristic high frequency, the other, much lower, as a control frequency that stabilizes the system. For instance, from the dynamical point of view a human body can provide an example of such a system. The frequency of the breath supports in a stable state the heart rhythm and deep slow breathing can stabilize the fast heart rhythm (see, for instance, [25] and references therein).

## ACKNOWLEDGMENTS

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